IRA A. FULTON SCHOOLS OF engineering

school of computing, informatics, & decision systems engineering

Introduction to Engineering Using Robotics Experiments

Numbering Systems and Binary

Dr. Yinong Chen

Contents

A Chinese Tale of a Numbering System

Once upon a time, there was a smart boy. His father wanted him to learn counting, so that he could better manage the family farm. The father sent him to a counting school

- On day one, he learned to write one: I
- On day two, he learned to write two: I I
- On day three, he learned to write three: I I I

The smart boy concluded that he did not need to attend the school anymore, because he was smart enough to figure out the rest of the numbers.

He needs to write the number 1024_{ten}

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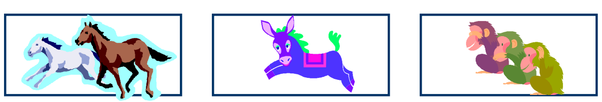
A numbering system in base-1 is invented

Non-Positional Numbering Systems

- Roman number system uses a non-positional scheme:
  I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, XIV, …, XLI
  - Addition is simple: XVI + VI = XVIVI = XXII (normalized)
  - Large number is very long
  - Multiplication and division are hard
- Akkadian system used in Babylonia around 2000 BC,



$60^2*2 + 60^1*1 + 60^0*3$

Each box (position) can have **several** symbols, representing 0 to 59, between boxes, it is positional, thus, forming a base 60 system

- A system in use in Greece from about 300 BC used a similar mixture. Each box can represent 0 to 999, and thus forming a base 1000 system

Positional Numbering Systems

http://www.psinvention.com/zoetic/basenumb.htm

- Base 1 (Unary)
- Base 2 (Binary)
- Base 5 (Hand)
- Base 8 (Octal)
- Base 10 (Decimal)
- Base 12 (duodecimal)
- Base 16 (Hexadecimal)
- Base 20 (Vigesimal)
- Base 60 (Sexagesimal)

Important in Engineering

- **Base 2 (Binary)**
- **Base 10 (Decimal)**
- **Base 16 (Hexadecimal)**

If you are a designer of a …
Can you design a …?

Math on 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 → Math on 0, 1

Binary

- Base 2: uses only 0's and 1's to represent all values
- Each place value represents 2^x, where x = 0, 1, 2, 3, …
- A binary number can have any number of leading zeroes: there is no impact on the value.
- 1's and 0's occupy the place values to indicate whether that value is turned "on" (1) or "off" (0)
- In digital systems, it is easy to use 1 for high voltage and 0 for low voltage.

Value of a Positional Number

Base 10 $2075_{ten} = 2 \cdot 10^3 + 0 \cdot 10^2 + 7 \cdot 10^1 + 5 \cdot 10^0$

Base 2 $1101_{two} = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$

$= 8 + 4 + 0 + 1$

Base *b* $d_{n-1} d_{n-2} \ldots d_2 d_1 d_0 = \sum_{i=0}^{n-1} d_i \cdot b^i$

where $d_i = 0, 1, \ldots, b-1$

Example: From Binary to Decimal

- **1 0 0 1 1 0 1 1**
  7 6 5 4 3 2 1 0 (position)

  10011011

  $= (1)(2^7) + (0)(2^6) + (0)(2^5) + (1)(2^4) + (1)(2^3) + (0)(2^2) + (1)(2^1) + (1)(2^0)$

  $= (1)(128) + (0)(64) + (0)(32) + (1)(16) + (1)(8) + (0)(4) + (1)(2) + (1)(1)$

The binary addition facts

$$\begin{array}{r} 0 \\ +\,0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ +\,0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 0 \\ +\,1 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +\,1 \\ \hline 10 \end{array}$$

Carry-in

		1	1	1	1	0	0	1	1	First number
		0	1	1	0	0	1	1	0	Second number
+	1	1	1	0	0	1	1	0	0	Carry-in
	1	0	1	0	1	1	0	0	1	Result

Decimal and Binary Comparison

Decimal	Binary	Decimal	Binary	Decimal	Binary
0	0	16	1 0000	32	10 0000
1	1	17	1 0001	33	10 0001
2	10	18	1 0010	34	10 0010
3	11	19	1 0011	35	10 0011
4	100	20	1 0100	36	10 0100
5	101	21	1 0101	37	10 0101
6	110	22	1 0110	38	10 0110
7	111	23	1 0111	39	10 0111
8	1000	24	1 1000	40	10 1000
9	1001	25	1 1001	41	10 1001
10	1010	26	1 1010	42	10 1010
11	1011	27	1 1011	43	10 1011
12	1100	28	1 1100	44	10 1100
13	1101	29	1 1101	45	10 1101
14	1110	30	1 1110	46	10 1110
15	1111	31	1 1111	47	10 1111

Decimal, Binary, and Hexadecimal

Decimal	Binary	Hex	Decimal	Binary	Hex	Decimal	Binary	Hex
0	0	0	16	1 0000	10	32	10 0000	20
1	1	1	17	1 0001	11	33	10 0001	21
2	10	2	18	1 0010	12	34	10 0010	22
3	11	3	19	1 0011	13	35	10 0011	23
4	100	4	20	1 0100	14	36	10 0100	24
5	101	5	21	1 0101	15	37	10 0101	25
6	110	6	22	1 0110	16	38	10 0110	26
7	111	7	23	1 0111	17	39	10 0111	27
8	1000	8	24	1 1000	18	40	10 1000	28
9	1001	9	25	1 1001	19	41	10 1001	29
10	1010	A	26	1 1010	1A	42	10 1010	2A
11	1011	B	27	1 1011	1B	43	10 1011	2B
12	1100	C	28	1 1100	1C	44	10 1100	2C
13	1101	D	29	1 1101	1D	45	10 1101	2D
14	1110	E	30	1 1110	1E	46	10 1110	2E
15	1111	F	31	1 1111	1F	47	10 1111	2F

From Binary to Decimal

The value of a binary number can be expressed by

$$\sum_{i=0}^{n-1} d_i \cdot 2^i$$

where $d_i = 0, 1$

$$2605_{ten} = 2 \cdot 10^3 + 6 \cdot 10^2 + 0 \cdot 10^1 + 5 \cdot 10^0$$

$$1101_{two} = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$= 8 + 4 + 0 + 1 = 13$$

1010111001010100

$$= 2^{15} + 2^{13} + 2^{11} + 2^{10} + 2^9 + 2^6 + 2^4 + 2^2$$

$$= 32768 + 8192 + 2046 + 1024 + 256 + 64 + 16 + 4$$

$$= 44628$$

From Decimal to Binary: Division Method

Convert decimal 94 to binary:

2	94	
2	47	0
2	23	1
2	11	1
2	5	1
2	2	1
2	1	0
	0	1

→ 1011110

To verify the correctness:

1011110_{two}

$= 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$

$= 64 + 0 + 16 + 8 + 4 + 2 + 0 = 94_{ten}$

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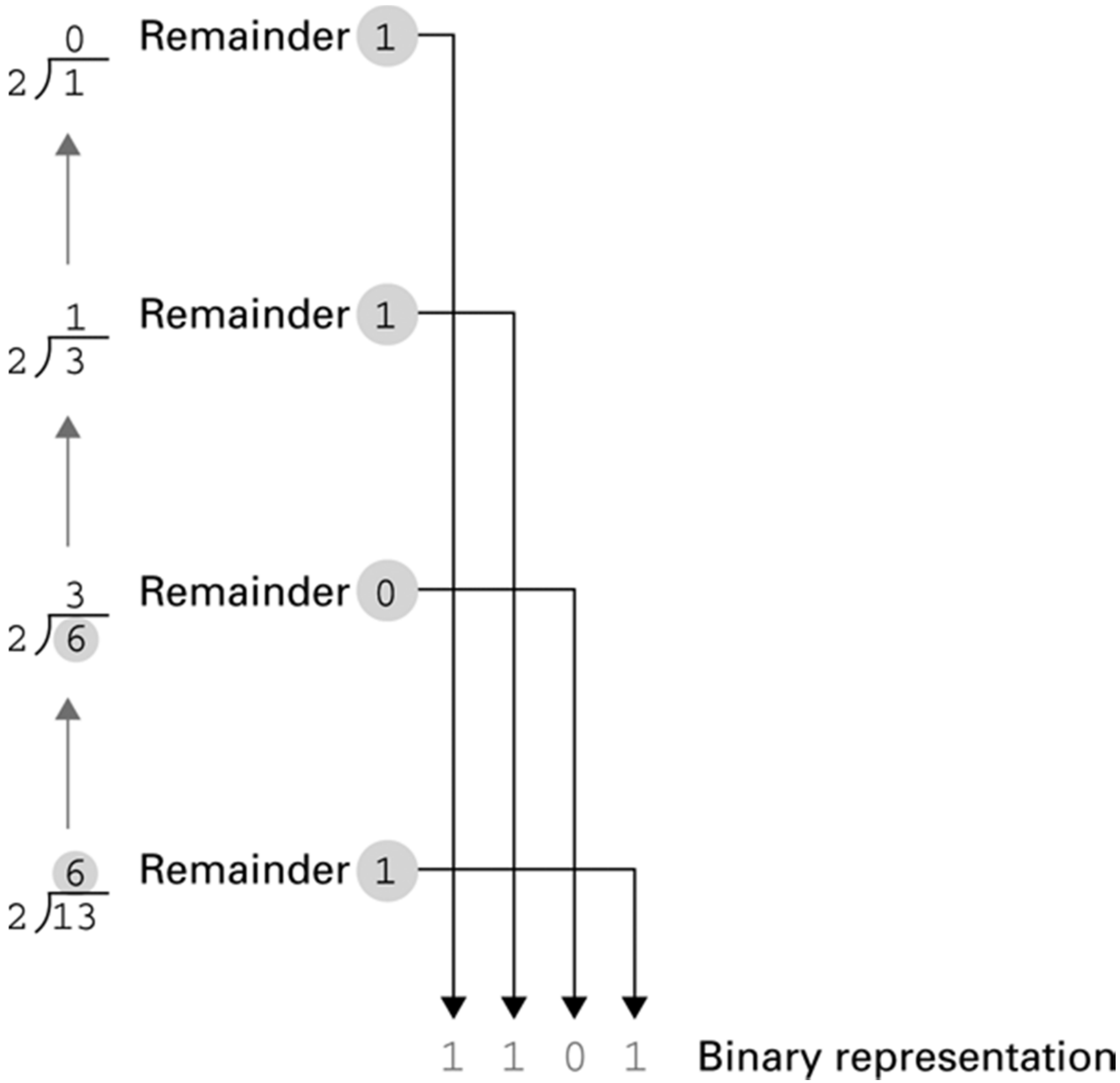

Between Binary and Hexadecimal

10 1010 1101 0011 0000 1111

2 A D 3 0 F → 2AD30F

3 B 4 F E 8 7 A 1 C D

11 1011 0100 1111 1110 1000 0111 1010 0001 1100 1101

Hexadecimal is most frequently used as a short hand notation of binary!